Mathematical Finance
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Recall 1

Process, modifications and indistinguishability

1. Let E be a subspace of a finite-dimensional normed vector space. We denote the Borel σ-algebra on E by $\mathcal{B}(E)$. What is an E-valued stochastic process?
2. What is a measurable process?
3. When do two E-valued processes X and Y have the same $\mathbb{P}$-finite-dimensional distribution? When are they $\mathbb{P}$-modifications of each other? When are they $\mathbb{P}$-indistinguishable?

Processes and filtrations

1. What are the left- and right-continuous limits $\mathcal{F}_{t-}$ and $\mathcal{F}_{t+}$ of a given filtration $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0}$?
2. What is an $\mathbb{F}$-adapted process?
3. Let X be an E-valued process and $\mathbb{F}$ be a filtration on $(\Omega, \mathcal{F}, \mathbb{P})$. What does it mean that X is $\mathbb{F}$-progressively measurable?
4. What are the *usual conditions* for a filtration $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0}$?
5. What is the *usual* $\mathbb{P}$*-augmentation* $\mathbb{F}^{\mathbb{P}+} := (\mathcal{F}_t^{\mathbb{P}+})_{t\geq 0}$ for a filtration $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0}$?

Processes and stopping times

1. What is a random time? What is an $\mathbb{F}$-stopping time? What is an $\mathbb{F}$-optional time?
2. Let G be a subset of E, what is the *first hitting time* of G for X? What it is the *début* of X in G? What is the *first approach time* of G for X?
3. Can you state some properties of stopping times?

Predictable and optional process

1. What is an $\mathbb{F}$-predictable stochastic process?
2. What is an $\mathbb{F}$-optional stochastic process?
3. Can you give one example for each of them?

Localisation

1. Let $\mathcal{X}$ be a family of processes. What is the localised class of $\mathcal{X}$, written $\mathcal{X}_{loc}(\mathbb{F}, \mathbb{P})$?
2. When is a class of processes $\mathcal{X}$ said to be $\mathbb{F}$-stable?
3. What does it mean that an E-valued process X is $(\mathbb{F}, \mathbb{P})$-locally bounded?
4. What does it mean that an E-valued process X is $(\mathbb{F}, \mathbb{P})$-locally integrable?

Specific convergence modes for stochastic processes

1. Can you state the uniform convergence in probability?
2. Can you show that the space of càdlàg $\mathbb{F}$-adapted processes is complete under the $\mathbb{P}$-ucp convergence?

The Émery topology

1. What does it mean that a sequence of càdlàg and $\mathbb{F}$-adapted processes $(X^n)_{n\in\mathbb{N}}$ converges to 0 under the $(\mathbb{F},\mathbb{P})$-Émery topology?